

All-optical routing of single photons with multiple input and output ports by interferences

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We propose a waveguide-cavity coupled system to achieve the routing of photons by the phases of other photons. Our router has four input ports and four output ports. The transport of the coherent-state photons injected through any input port can be controlled by the phases of the coherent-state photons injected through other input ports. This control can be achieved when the mean numbers of the routed and control photons are small enough and require no additional control fields. Therefore, the all-optical routing of photons can be achieved at the single-photon level.

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Quantum network [1] plays an essential role in quantum information and quantum computation [2]. The routing capability of information is a requisite in quantum network. Photons are considered as the ideal carrier of quantum information. Therefore, the investigation of all-optical routing of photons at the single-photon level will have direct application to realize quantum networks for optical quantum information and quantum computation. Recently, a scheme to achieve all-optical routing of single photons with two input ports and two output ports has been demonstrated [3]. In their scheme, the control single photons and routed single photons are connected by an intermediate three-level atom. By coupling two different atomic transitions, respectively, to the routed and control photons, the routed single photons can be controlled through injecting the control single photons. Currently, it will be of interest to realize the all-optical routing of photons at the single-photon level in other physical mechanism. Moreover, the all-optical routing with more than two input and output ports, which is essential for the quantum network, still needs to be explored.

For these purposes, we propose a scheme to study the all-optical routing of coherent-state photons with four input ports and four output ports by other coherent-state photons. It is significant that the all-optical routing of photons is realized by the interferences depending on the phase differences between the routed and the control photons. Our scheme is based on the waveguide QED system [4–20], in which the strong coupling between the waveguide photons and the emitters coupled to the waveguide is realized. The routed photons and control photons are connected by an intermediate single-mode cavity. When the photons in the coherent state are injected into any of the input ports, the photon transport does not depend on the phase of the photons. However, when more than one

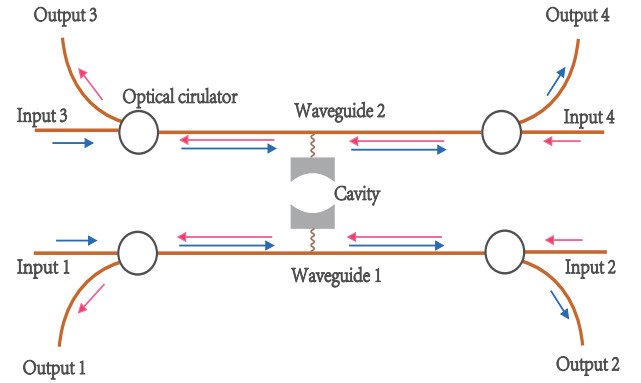


FIG. 1: Schematic configuration of the all-optical routing of single photons with four input ports and four output ports. The two optical waveguides are connected by an optical cavity. Four optical circulators are employed to separate the input ports from output ports.

input ports are injected with coherent-state photons, the photon transport can be controlled by the phase differences between the photons injected into different ports. The routed photons and control photons have the equal mean photon numbers and frequencies. Consequently, the routed photons can act as the control photons and the control ones can act as the routed ones. In our router, the mean photon numbers can be either small or large. Therefore, our router can be realized at the single-photon level. Under certain conditions, our scheme is a router with two input ports and two output ports. Compared to [3], the intermediate single-mode cavity is coupled to both the routed and control photons in our scheme. This may avoid the cross-contamination of matching different atomic transitions, respectively, to the routed and control photons.

The system under consideration is depicted in Fig. 1.

The cavity is strongly side-coupled to lossless waveguide 1 and 2. The right (left)-moving photons in waveguide 1 are connected to the input port 1 (2) and output port 2 (1) with the optical circulators. And the right (left)-moving photons in waveguide 2 are connected to the input port 3 (4) and output port 4 (3). The photons injected into any of the input ports move along the 1D waveguides and then are scattered due to the photon-cavity interaction. After scattering, the photons may be redirected. Here we focus on the photon transport influenced by the photon phases. The system Hamiltonian in the rotating-wave approximation is written as ($\hbar = 1$)

$$H = \sum_{j=1,2} \left(\int d\omega \omega r_{j\omega}^\dagger r_{j\omega} + \int d\omega \omega l_{j\omega}^\dagger l_{j\omega} \right) + \omega_c c^\dagger c \quad (1)$$

$$+ \sum_{j=1,2} \int d\omega g_j c^\dagger (r_{j\omega} e^{i\omega z_c/v_g} + l_{j\omega} e^{-i\omega z_c/v_g}) + h.c.,$$

where $r_{j\omega}^\dagger$ ($l_{j\omega}^\dagger$) creates a right(left) propagating photon with frequency ω in the waveguide j , c^\dagger creates a photon in the cavity, ω_c is the cavity resonance frequency, g_j is the coupling strength of the cavity to the waveguide j , z_c is the position of the cavity, and v_g is the group velocity of the photons. Here, we have assumed that g_j is frequency-independent, which is equivalent to the Markovian approximation. The waveguides are considered with the linear dispersion relation, i.e. $\omega = v_g |k|$, with k wave number. We will take z_c zero and extend the frequency integration to $\pm\infty$ below.

We study the photon scattering with input-output formalism [21]. The input and output operators are defined as $o_j^{(in)}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega o_{j\omega}(t_0) e^{-i\omega(t-t_0)}$ ($o = r, l$) and $o_j^{(out)}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega o_{j\omega}(t_1) e^{-i\omega(t-t_1)}$, respectively. The operator $o_{j\omega}^{(in)}$ and $o_{j\omega}^{(out)}$ in the scattering theory are related to the input and output operators through $o_j^{(in)}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega o_{j\omega}^{(in)} e^{-i\omega t}$ and $o_j^{(in)}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega o_{j\omega}^{(in)} e^{-i\omega t}$ [14], respectively. The system initial state $|\Psi_0\rangle$ is a simple product state of the two waveguide field states and the cavity state. In our scheme, initially, the cavity is in the vacuum state and the injected photons are in the coherent states. For the coherent input state, $o_j^{(in)}(t) |\Psi_0\rangle = \frac{1}{\sqrt{2\pi}} \int d\omega \alpha_\omega e^{-i\omega t} |\Psi_0\rangle$, with α_ω being a complex number. The mean number of the coherent-state photons is represented by $\int d\omega |\alpha_\omega|^2$. By the input-output formalism, we find

$$o_j^{(out)}(t) = o_j^{(in)}(t) - i\sqrt{\gamma_j} c(t), \quad (2)$$

$$\dot{c}(t) = (-i\omega_c - \sum_j \gamma_j) c(t) - i \sum_{j,o} \sqrt{\gamma_j} o_j^{(in)}(t),$$

with $\gamma_j = 2\pi g_j^2$ being the decay rates from the cavity to the waveguide j . From Eqs. (2), both the expectation values $\langle \Psi_0 | o_j^{(out)\dagger}(t) o_j^{(out)}(t) | \Psi_0 \rangle$ and

$\langle \Psi_0 | o_{j\omega}^{(out)\dagger} o_{j\omega}^{(out)} | \Psi_0 \rangle$ can be obtained under the initial conditions.

We first consider the case that the photons with frequency ω in a coherent state with mean photon number $|\alpha|^2$ are injected into input port 1. After calculations, we obtain $o_j^{(out)}(t) |\Psi_0\rangle = f_{oj}(\gamma_1, \gamma_1, \delta_\omega, \omega_c, \omega) e^{-i\omega t} |\Psi_0\rangle$. Therefore, the output photons have the same frequency with the input photons due to the conservation of energy. The mean numbers of the photons outputting from each ports are

$$N_{r1}^{(out)} = \frac{\delta_\omega^2 + \gamma_2^2}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2, \quad (3)$$

$$N_{l1}^{(out)} = \frac{\gamma_1^2}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2,$$

$$N_{r2}^{(out)} = N_{l2}^{(out)} = \frac{\gamma_1 \gamma_2}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2,$$

with $\delta_\omega = \omega_c - \omega$ the detuning. $N_{rj}^{(out)}$ ($N_{lj}^{(out)}$) is the mean number of the right (left)-moving photons in the waveguide j after scattering. Hence, $N_{r1}^{(out)}$, $N_{l1}^{(out)}$, $N_{r2}^{(out)}$ and $N_{l2}^{(out)}$ correspond to the mean numbers of the photons outputting from ports 2, 1, 4 and 3, respectively. It is easy to verify the conservation relation $\sum_{o,j} N_{oj}^{(out)} = \langle r_1^{(in)\dagger}(t) r_1^{(in)}(t) \rangle = |\alpha|^2$. When the input photons resonantly interact with the cavity and the coupling strengths of the cavity to the two waveguides are equal, i.e. $\delta_\omega = 0$ and $\gamma_1 = \gamma_2$, the photons are redirected into the four output ports equally. When $\delta_\omega \gg \gamma_1$ or $\gamma_2 \gg \gamma_1$, the waveguide 1 is almost decoupled to the cavity and we find $N_{r1}^{(out)} \rightarrow |\alpha|^2$. When the cavity is decoupled to the waveguide 2 and the input photons resonantly interact with the cavity, the photons are completely reflected and redirected into the output port 1.

The photons injected into different ports arrive at the position z_c simultaneously and then interact with the intermediate cavity. We proceed to study the routing of the photons by photons in two cases. One case is the routing of photons by photons injected into another input ports, the other case is by photons injected into other two input ports.

Two-input case.—In the two-input case, it is enough to study the situations that photons are injected into port 1 and 2 and that the photons are injected into port 1 and 3. This can be understood by the expression of Hamiltonian (1). When the photons in the coherent states are injected into port 1 and 2, the mean numbers of the output photons are obtained as

$$N_{r1}^{(out)} = [1 - 2 \frac{(1 + \cos \phi) \gamma_1 \gamma_2 + \gamma_1 \delta_\omega \sin \phi}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2}] |\alpha|^2, \quad (4)$$

$$N_{l1}^{(out)} = [1 - 2 \frac{(1 + \cos \phi) \gamma_1 \gamma_2 - \gamma_1 \delta_\omega \sin \phi}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2}] |\alpha|^2,$$

$$N_{r2}^{(out)} = N_{l2}^{(out)} = \frac{2(1 + \cos \phi) \gamma_2 \gamma_1}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2,$$

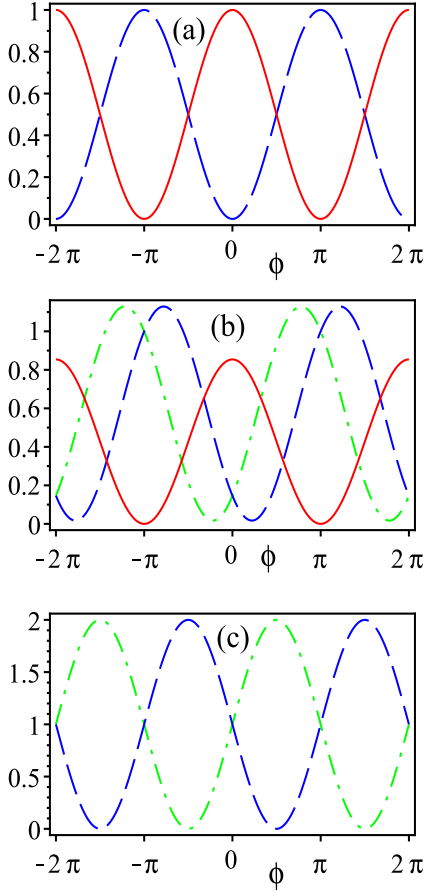


FIG. 2: The mean photon numbers $N_{oj}^{(out)}$ against the phase differences θ in the two-input case. The blue dashed lines are $N_{r1}^{(out)}$, the green dashed dotted lines are $N_{l1}^{(out)}$, the red dotted lines are $N_{r2}^{(out)} = N_{l2}^{(out)}$. We take $\delta_\omega = 0$, $\gamma_2 = \gamma_1$ in (a), $\delta_\omega = 0.5\gamma_1$, $\gamma_2 = 0.6\gamma_1$, in (b), and $\delta_\omega = \gamma_1$, $\gamma_2 = 0$ in (c). The other parameter is $|\alpha|^2 = 1$. All the parameters but $|\alpha|^2$ are in units of γ_1 . The mean photon number $N_{l1}^{(out)}$ is not given in (a) because $N_{r1}^{(out)} = N_{l1}^{(out)}$.

with ϕ being the phase difference between the photons injected into different ports. Here we have taken that the photons injected into the two input ports have the same photon mean number $|\alpha|^2$ and the same frequency ω . Similar to the single-input case, the output photons have the same frequency with the input photons. It is interesting that the expressions of the mean output-photon numbers are periodic functions of ϕ with period 2π . Therefore, the routing of photons can be achieved by the phase of other photons injected into another input port. When $\phi = 2\pi$, $\delta_\omega = 0$ and $\gamma_1 = \gamma_2$, the photons are completely redirected into output ports 3 and 4 due to the constructive interference. However, when $\phi = \pi$, the photons are completely redirected into output ports 1 and 2 due to the destructive interference. To see the details of the routing property, we plot the mean photon numbers in Eqs. (3) against the phase difference in

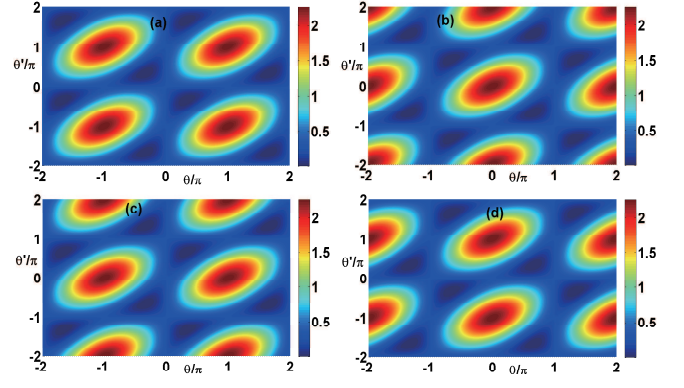


FIG. 3: The mean photon numbers against the phase differences in the three-input case. (a), (b), (c) and (d) denote $N_{r1}^{(out)}$, $N_{l1}^{(out)}$, $N_{r2}^{(out)}$ and $N_{l2}^{(out)}$, respectively. For all the plots, the parameters are $\delta_k = 0$, $\gamma_2 = \gamma_1$, and $|\alpha|^2 = 1$. All the parameters but $|\alpha|^2$ are in units of γ_1 .

fig. 2. Therefore, the routing of the coherent-state photons injected into the input port 1 can be achieved by the phase of the coherent-state photons injected into the input port 2. In our scheme, this routing is based on the interferences determined by the phase difference. These interferences can not be obtained when the input photons are in Fock states [16]. This is because the coherent state is the eigenstate of the annihilation operator.

When the cavity is decoupled to the waveguide 2, i.e. $\gamma_2 = 0$, our scheme becomes a router with two input and two output ports. The mean numbers of the photons outputting from either port are obtained as $N_{r1}^{(out)} = \frac{\delta_\omega^2 + \gamma_1^2 - 2\gamma_1 \sin \phi \delta_\omega}{\delta_\omega^2 + \gamma_1^2} |\alpha|^2$, and $N_{l1}^{(out)} = \frac{\delta_\omega^2 + \gamma_1^2 + 2\gamma_1 \sin \phi \delta_\omega}{\delta_\omega^2 + \gamma_1^2} |\alpha|^2$. It is interesting that when $\delta_\omega^2 = \gamma_1^2$, the expectation value $N_{o1}^{(out)}$ can be from 0 to $2|\alpha|^2$ by adjusting the phase ϕ . The details are shown in Fig. 2c.

When the photons in coherent states are injected into input ports 1 and 3, the outcomes have the forms similar to the outcomes in Eqs. (4) except γ_j . Hence, it is not necessary to study the details of this situation. When $\gamma_1 = \gamma_2$, the outcomes are equal to the outcomes in Eqs. (4). Consequently, under the conditions $\gamma_1 = \gamma_2$ and $\delta_\omega = 0$, the photons can be completely directed into output ports 2 (1) and 4 (2) when $\phi = \pi$ ($\phi = 2\pi$).

Three-input case.—In the three-input case, it is enough to study the situation that the photons are injected into input ports 1, 3 and 4. When the coherent-state photons with frequency ω are injected into the input ports 1, 3 and 4, the output photons have the frequency ω and the mean numbers of the output photons are obtained as

$$\begin{aligned}
N_{r1}^{(out)} &= \frac{\delta_\omega^2 + \gamma_2^2 + 2\gamma_1\gamma_2 - 2\delta_\omega\sqrt{\gamma_1\gamma_2}(\sin\theta + \sin\theta') - 2\sqrt{\gamma_1\gamma_2}\gamma_2(\cos\theta + \cos\theta') + 2\cos(\theta - \theta')\gamma_1\gamma_2}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2, \quad (5) \\
N_{l1}^{(out)} &= \frac{\gamma_1^2 + 2[1 + \cos(\theta - \theta')]\gamma_1\gamma_2 + 2(\cos\theta + \cos\theta')\gamma_1\sqrt{\gamma_1\gamma_2}}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2, \\
N_{r2}^{(out)} &= \frac{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2 - \gamma_2\gamma_1 + 2\sin\theta\sqrt{\gamma_1\gamma_2}\delta_\omega + 2\sin(\theta - \theta')\gamma_2\delta_\omega - 2\cos\theta\gamma_1\sqrt{\gamma_1\gamma_2} + 2\cos\theta'\gamma_2\sqrt{\gamma_1\gamma_2} - 2\cos(\theta' - \theta)\gamma_1\gamma_2}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2, \\
N_{l2}^{(out)} &= \frac{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2 - \gamma_2\gamma_1 + 2\delta_\omega[\sin\theta'\sqrt{\gamma_1\gamma_2} + \sin(\theta' - \theta)\gamma_2] - 2\cos(\theta - \theta')\gamma_2\gamma_1 - 2\cos\theta'\gamma_1\sqrt{\gamma_1\gamma_2} + 2\cos\theta\gamma_2\sqrt{\gamma_1\gamma_2}}{\delta_\omega^2 + (\gamma_1 + \gamma_2)^2} |\alpha|^2,
\end{aligned}$$

with θ (θ') the phase difference between the photons injected into input ports 1 and 3 (4). The photons injected into each of the three ports have the same mean photon number $|\alpha|^2$. The mean numbers of output photons in Eqs. (5) against the phase differences are plotted in Fig. 3. It shows that the routing of the photons by other photons can be achieved in the three-input case.

We note that although we have taken the mean photon number $|\alpha|^2 = 1$ in all the plots, the routing properties do not depend on $|\alpha|^2$. This can be understood from the expressions of Eqs. (4) and (5). Hence, the routing can be achieved at the single-photon level.

We have studied the routing of photons when the input photons are in single-mode coherent states, without considering the cavity decay to other modes but the waveguide modes. The cavity decay can be incorporated by introducing the nonhermitian Hamiltonian $H_{non} = -i\gamma_c c^\dagger c$, with γ_c the decay rate. The injected coherent-state prepared in a Gaussian-type wave packet is defined as $a_\omega^{(in)} |\Psi_0\rangle = \alpha_\omega |\Psi_0\rangle$. The complex number α_k has the form of $\alpha_\omega = \frac{\alpha}{\sqrt{2\pi\Omega^2}} e^{-\frac{(\omega-\omega_0)^2}{4\Omega^2}}$, with 2Ω the bandwidth and ω_0 the center frequency. The mean photon number of the wave packet is $\int d\omega |\alpha_\omega|^2 = |\alpha|^2$. For the Gaussian-type wave-packet input, the mean output-photon numbers can be obtained by numerical evaluations. We plot the routing property when the photons in the coherent state prepared in Gaussian-type wave packet are injected into input ports 1 and 2 in Fig. 4. In Fig. 4, the cavity decay has been incorporated. In Fig. 4(a), the up bound of $N_{r1}^{(out)} = N_{l1}^{(out)}$ is barely affected but the up bound of $N_{r2}^{(out)} = N_{l2}^{(out)}$ decreases evidently compared to Fig. 2. In Fig. 4(c), both the up bound $N_{r1}^{(out)}$ and $N_{l1}^{(out)}$ decrease evidently. These are mainly due to the fact that we have considered the wave-packet bandwidth, which can be understood as follows. The frequency-dependent condition $\delta_\omega = 0$ is necessary when the value

of $N_{r2}^{(out)}$ in Fig. 4(a) reaches unit. However, the unit value of $N_{r1}^{(out)}$ in 4(a) only needs the condition $\theta = \pi$, which is frequency-independent. In 4(c), we take $\delta_\omega = \gamma_1$, which is frequency-dependent. The outcomes obtained under the frequency-dependent condition are affected by the bandwidth. The effect caused by the cavity decay can be studied by the mean number $N^{(out)}$ of all the output photons, with $N^{(out)} = N_{r1}^{(out)} + N_{l1}^{(out)} + N_{r2}^{(out)} + N_{l2}^{(out)}$. As is shown in Fig. 4(d), when $\theta = \pi$, the $N^{(out)}$ is not affected by the cavity decay due to the destructive interference. However, when $\theta = 2\pi$, the cavity decay has obvious effect due to the constructive interference.

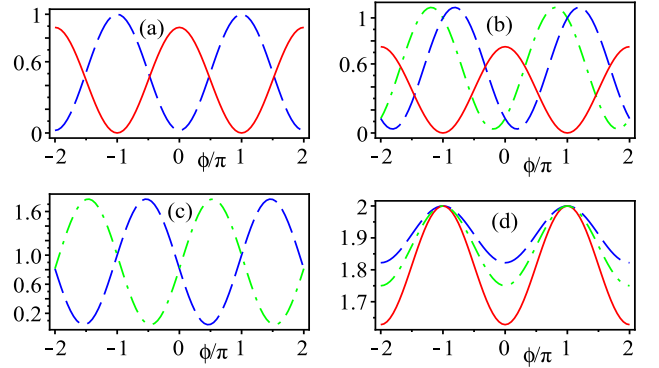


FIG. 4: The mean numbers of the output photons against the phase differences in the two-input case. The input photons are in coherent states prepared in Gaussian-type wavepackets and the cavity decay has been considered. For all the plots, we take $\Omega = 0.3\gamma_1$ and $\gamma_c = 0.1\gamma_1$. In (a), (b) and (c), the blue dashed lines are $N_{r1}^{(out)}$, the green dashed dotted lines are $N_{l1}^{(out)}$, the red dotted lines are $N_{r2}^{(out)} = N_{l2}^{(out)}$. The other parameters in (a), (b) and (c) are the same to the parameters in Fig. 2(a), 2(b) and 2(c), respectively. (d) shows the sum of the mean numbers of the photons outputting from all output ports $N^{(out)}$. The blue dashed line, green dashed dotted line, and red solid lines in (d) correspond to the situations as shown in (a), (b) and (c), respectively.

In conclusion, we have presented a detailed investigation on the routing of single photons with four input ports and four output ports by single photons. The routing is achieved by the interferences related to the phase differences between the coherent-state photons. The routed photons can play the role of control photons, and the control photons can also play the role of routed photons. Our scheme is of significance to build the quantum networks. We hope that this routing will be achieved experimentally in the near future.

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